

## System Configuration And Processing Method For Forward Looking SAR With Two Receiving Antennas

Shengli Dai<sup>1</sup> Werner WIESBECK (Fellow IEEE)<sup>2</sup>

<sup>1</sup>Dept. Electronic Eng., Jinan University, 510632 Guangzhou, China  
Tel: 86-20-85220483, Fax: 86-20-85221941  
E-mail: tsld@jnu.edu.cn

<sup>2</sup>IHE, University Karlsruhe, Kaiserstr. 12, 76128 Karlsruhe, Germany  
Tel: 49-721-6082522, Fax: 49-721-691865  
E-mail: werner.wiesbeck@etec.uni-karlsruhe.de

**ABSTRACT** Compared to side looking Synthetic Aperture Radar (SAR), forward looking SAR (F-SAR) is known to have the disadvantage of the left/right ambiguity problem and difficult to achieve high angular resolution (azimuth resolution). In this paper, a new F-SAR mode with two receiving antennas and one transmitting antenna on the radar platform is proposed. In this operation mode, the left/right ambiguity problem can be solved in theory from the view of geometry relationship. High angular (azimuth) resolution can be achieved by using a new compression method in a new receiving antennas in one synthetic aperture time with a proposed method. The imaging mode, the azimuth resolution and the azimuth reference matching filter function are analyzed. Finally simulation results are provided.

**KEY WORDS** Forward looking, SAR imaging, azimuth compression, left/right ambiguity, Doppler-time domain.

### 1. Introduction

In some applications a forward looking imaging radar is desirable, e.g. for aircraft landing systems and surveillance. Forward looking SAR (F-SAR) can obtain high resolution images of the imaged area directly in front of the radar platform in theory. But there are some new key techniques requirements that should be solved in theory and in practice.

In this paper the angular resolution, known as azimuth resolution, is perpendicular to the moving direction of radar platform, and the slant range is along the moving direction of the radar platform. The reasons for low azimuth resolution and the left/right ambiguity problems are that terrain targets located symmetrically along side the moving path have the same Doppler history to one antenna and the gradient of the Doppler frequency of targets is very small in the moving direction of the radar platform. For the forward looking SAR, many research papers and work are based on the imaging mode with only one antenna [1,2]. In order to improve the azimuth resolution, the forward looking radar of F-SAR should work in the Doppler beam sharpening (DBS) mode or with special sophisticated antenna illumination functions. Some papers proposed processing methods using minimum variance unbiased (MVU) estimation which leads to undesirable noise enhancement. To minimize the noise enhancement and thus maximize the angular resolution, antennas with specific aperture illumination functions are proposed and zero forcing (ZF) estimation is adopted.

This paper considers a radar platform with two

receiving antennas and one transmitting antenna (one receiving antenna can be used for transmission). The line joining the two receiving antennas is perpendicular to the moving direction of the radar platform. The transmitting antenna is also on this line. In this mode, the ambiguity problem of the one antenna radar imaging mode can be solved from the Doppler histories of targets for the two receiving antennas. If the radar transmits FM signals at a regular pulse repetition frequency (PRF) while the radar platform is moving forward, each time two sets of equal-range line data can be acquired from the two receiving antennas after range compression. Consequently, two synthetic apertures are obtained after one synthetic aperture time along the azimuth direction by the two receiving antennas. These can be used for high resolution imaging and solving left/right ambiguity.

This paper deals with three main difficulties of F-SAR: a) how to design the system configuration for this mode. b) how to generate the matched filter function for azimuth compression to get high resolution. c) how to set the data of the two receiving antennas properly and use suitable processing to solve the left/right ambiguity problem efficiently.

### 2. System Configuration and Analysis

The basic geometry of the forward looking radar or F-SAR system is shown in Fig.1. The effect of the incidence angle in elevation is not considered here. The  $y$  direction is the flight direction of radar platform., and  $x$  is the azimuth direction.  $R_{x1}$  and  $R_{x2}$  are two receiving antennas.  $T$  is the transmitting antenna which can lie outside or inside the line

joining the two receiving antennas, but is assumed on the extension of the joining line in the following analysis. The distances between  $R_{x1}$  and  $R_{x2}$ ,  $R_{x1}$  and  $T$ , and  $T$  and  $R_{x2}$  are  $l$ ,  $l_1$  and  $l_2$ , respectively. Here  $l=l_1+l_2$ ,  $v$  is the velocity of radar platform, and  $y$  is defined by  $y=vt$ . The point  $(x_0, r_0)$  is any target at azimuth position  $x_0$  in the illumination area of the beam footprints of the transmitting and receiving antennas within the range gate  $r_0$ .

From Fig.1(a), one can see that there is the ambiguity problem if the antenna is fixed (not scanning) one the radar platform for the forward looking radar with one receiving antenna. The ambiguity problem is solved in Fig.1(b) from the view of geometry relationship between the receiving antennas  $R_{x1,2}$  and the target. Although there are some problems in practice, such as that the length of targets along azimuth will affect the elimination results to the ambiguity. This will be analyzed in the following section. Fig.1(c) shows that there are two symmetry lines to  $R_{x1}$  and  $R_{x2}$  respectively. The whole imaging area ( or the whole equal range gate) is separated into three sub-sections, sub-section I, II, III in the ambiguity elimination processing by using the data from two receiving antennas.

It is assumed that the transmitter is at the coordinate  $(x, y)=(0,0)$  at  $t=0$ . The slant range history of the target  $(x_0, r_0)$  relative to  $R_{x1}$  and  $R_{x2}$  is expressed by 1(a) and 1(b), respectively

$$r_1(t, x_0, r_0) = \sqrt{(r_0 - vt)^2 + x_0^2} + \sqrt{(r_0 - vt)^2 + (x_0 + l_1)^2} \quad (1a)$$

$$r_2(t, x_0, r_0) = \sqrt{(r_0 - vt)^2 + x_0^2} + \sqrt{(r_0 - vt)^2 + (x_0 - l_2)^2} \quad (1b)$$

Using Taylor series to expand (1a):

$$r_1(t, x_0, r_0) = (R_0 + R_1) - \left(\frac{r_0}{R_0} + \frac{r_0}{R_1}\right)vt + \frac{1}{2} \left(\frac{1}{R_0} + \frac{1}{R_1} - \frac{r_0^2}{R_0^3} - \frac{r_0^2}{R_1^3}\right)(vt)^2 + \frac{1}{2} \left(\frac{1}{R_0^3} + \frac{1}{R_1^3} - \frac{r_0^3}{R_0^5} - \frac{r_0^3}{R_1^5}\right)(vt)^3 + \dots \quad (2)$$

where

$$R_0 = \sqrt{r_0^2 + x_0^2}, \quad R_1 = \sqrt{r_0^2 + (x_0 + l_1)^2}, \quad R_2 = \sqrt{r_0^2 + (x_0 - l_2)^2} \quad (3)$$

For a target  $(0, r_0)$  at center of the imaging area, the slant range history from  $R_{x1}$  is

$$r_1(t, x_0, r_0) = (r_0 + r_1) - \left(1 + \frac{r_0}{r_1}\right)vt + \frac{1}{2} \left(\frac{1}{r_1} - \frac{r_0^2}{r_1^3}\right)(vt)^2 + \frac{1}{2} \left(\frac{r_0}{r_1^3} - \frac{r_0^3}{r_1^5}\right)(vt)^3 + \dots \quad (4)$$

$$\text{where } r_1 = \sqrt{r_0^2 + l_1^2}$$

From the previous equations, the slant range (Doppler or phase) histories of targets are dependent on the distance  $y$ , the reference range  $r_0$  and the coordinate  $x_0$  of the target.

The Doppler frequency history  $f_{D1}(t, x_0, r_0)$  of the target to  $R_{x1}$  can be obtained from (2) with  $dr_1(t, x_0, r_0)/dt$  as

$$f_{D1}(t, x_0, r_0) = \frac{1}{\lambda} \left[ -\left(\frac{r_0}{R_0} + \frac{r_0}{R_1}\right)v + \left(\frac{1}{R_0} + \frac{1}{R_1} - \frac{r_0^2}{R_0^3} - \frac{r_0^2}{R_1^3}\right)v^2 t + \frac{3}{2} \left(\frac{r_0}{R_0^3} + \frac{r_0}{R_1^3} - \frac{r_0^3}{R_0^5} - \frac{r_0^3}{R_1^5}\right)v^3 t^2 + \dots \right] \quad (5)$$

where  $\lambda$  is wave length.

The similar analysis and expanding are applied to antenna  $R_{x2}$ , and will render similar results.

It shows that the Doppler frequency of a target is very large ( the linear term of  $v$ ) while the gradient of the Doppler frequency ( the 2nd and higher terms of  $v$ ) is relatively small. This is opposite compared to the classic side looking SAR in which the Doppler frequency is almost 0, and the gradient of Doppler frequency is relatively high. Therefore it is impossible to get a high azimuth resolution for F-SAR by using a similar azimuth compression method like in classic side looking SAR. A new azimuth compression method is proposed in the following for obtaining a high azimuth resolution.

### 3. Azimuth Compression Method for F-SAR

For a high range resolution SAR, the traditional transmitting signal is a FM signal, given by:

$$s_t(t) = a(t)e^{j2\pi f_c t} e^{j\pi k_f t^2} \quad (6)$$

where  $a(t)$  is the envelope of transmitted signal,  $f_c$  is the carrier frequency, defined by  $f_c = c/\lambda$ ,  $c$  is the velocity of light,  $k_f$  is the FM rate.

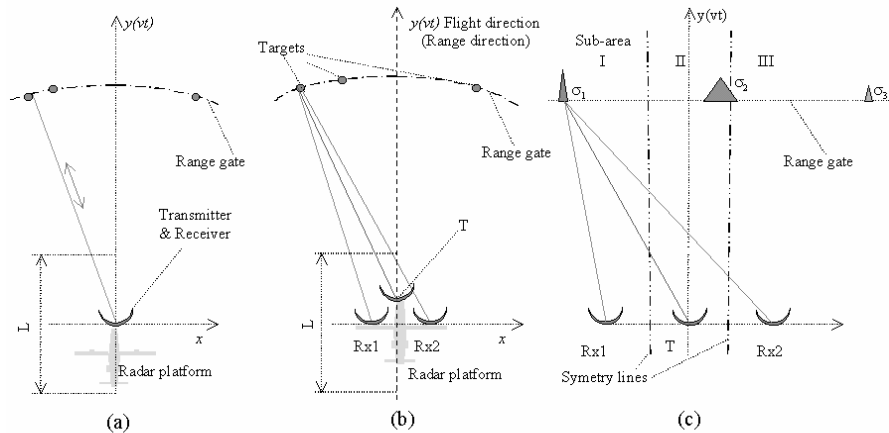


Fig.1 Geometry of forward looking radar or F-SAR: (a) with only one receiving antenna, (b) and (c) with two receiving antennas and one transmitting antenna

After the range compression of the received signal at each discrete receiving point along the flight path, the range compressed signals are stored for one aperture time for azimuth compression processing. Since the range compression is well known [3], the azimuth compression in a single range gate is only considered in the following analysis. It is also assumed that the range migration and curve and motion of the radar platform have been compensated perfectly, and the footprints of the two receiving antennas and the transmitting antenna cover the same imaging area at any time. The range compressed signal can be briefly expressed as:

$$s(\tau, t) = \psi(\tau) e^{-j2\pi f_c \frac{r(t)}{c}} \quad (7)$$

The azimuth compression is to compress the signal of (7) with a matched azimuth filter function in one section 2, it is obvious that the targets located at different azimuth positions within the same range gate have different Doppler histories in the F-SAR mode. So the different matched azimuth reference functions should be adopted to the targets at different azimuth positions.

First the Doppler histories of F-SAR are compared with that of classic side looking SAR, shown in Fig.2. the two axes are  $f$  (Doppler frequency) and  $t$  (synthetic time).  $L$  is the length of the synthetic aperture. Fig.2(a) shows the Doppler histories of targets for the side looking SAR, and Fig.2(b) shows the Doppler histories of targets for the F-SAR at different azimuth position within the same range gate  $r_0$ . To the side looking SAR, the traditional matched filter can be applied to compress the Doppler histories in the time-Doppler ( $t$ - $f$ ) domain shown in Figs 2(a) and (b). The high azimuth resolution can be obtained for classic side looking SAR. But for the F-SAR, it is very difficult to obtain a high azimuth resolution directly by using a matched filter to compress the Doppler histories in time-Doppler ( $t$ - $f$ ) domain. The format of Doppler frequency histories of these two kinds of imaging mode are very different. If the Doppler frequency

histories are expressed in another format, in Doppler-time ( $f$ - $t$ ) domain shown in Fig.2(c), one can see the format of Doppler frequency history of targets in F-SAR is similar to that of the classic side looking SAR, although with some differences. So the high azimuth resolution may be obtained by using a suitable compressing method with their matched filters that works in Doppler-time ( $f$ - $t$ ) domain.

For the F-SAR, the Doppler histories of targets at different azimuth position are different, the different matched reference functions are needed to compress the Doppler histories to get perfect compressed results in theory. In order to save computation time in practice, the whole raw data in one aperture time frame from two receiving antennas can be separated into many sub-sections in Doppler frequency domain in the first step, and then generate matched reference function for each sub-section together to obtain the whole compressed results are performed.

#### 4. Solving the Left/Right Ambiguity

For the left/right ambiguity problem, fig.1(c) shows how and where it comes from in the imaging mode with two receiving antennas. In fig.1(c),  $\sigma_i$  is the radar cross section (RCS) of target  $i$ , the whole raw data in one range gate is separated into three sub-sections (sub-areas) with respect to the two symmetry lines. It is assumed that targets 1, 2 and 3 are the only three targets within the same equal-range gate.  $\sigma_1$  and  $\sigma_2$  are symmetrical to  $R_{x1}$ , and  $\sigma_1$  and  $\sigma_3$  are symmetrical to  $R_{x2}$ . The raw data are obtained from these targets after one aperture time  $L/v$  as shown in fig.3 in Doppler-time ( $f$ - $t$ ) domain.  $R_{x1}$  and  $R_{x2}$  mean that the data are obtained from receiving antennas 1 and 2 in fig.3, respectively. And  $\Delta f$  means the frequency difference between  $\sigma_1$  and  $\sigma_3$  for the same antenna  $R_{x1}$ . It is relative to azimuth resolution.  $\delta f$  means the frequency difference (shifting) of the same  $\sigma_1$  at the two receiving antennas  $R_{x1}$  and  $R_{x2}$ . After a suitable

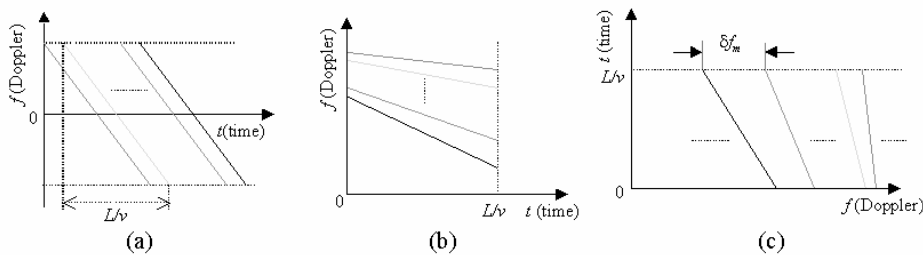


Fig.2 Doppler histories in one synthetic aperture time: (a) the Doppler histories of side looking SAR, (b) and (c) the Doppler histories of F-SAR

azimuth compression, proposed in the above section, the high resolution results shown in Fig.3(b) are acquired.

Till now, the left/right ambiguity problem has not been solved. But if the azimuth compressed data of  $R_{x2}$  in fig.3(b) is shifted to the right direction, and  $\sigma_1+\sigma_2$  and  $\sigma_1+\sigma_3$  are made to coincide in the same position. The result of  $\sigma_1$  is obtained after multiplication and square root calculation, expressed in Fig.3(c) and equ.(8)

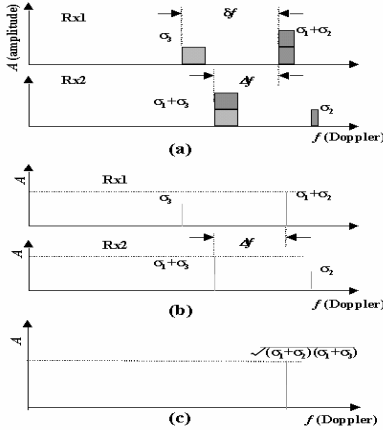


Fig.3 The ambiguity problem and its solution: (a) the raw data of three targets in Doppler domain, (b) the compression results of data from (a), (c) the result after solving the ambiguity of target 1.

if  $\sigma_1 \gg \sigma_2, \sigma_3$ . It often happens in practice.

$$\sigma_1 = \sqrt{(\sigma_1+\sigma_2)(\sigma_1+\sigma_3)} \quad (8)$$

If  $\sigma_1 \approx \sigma_2, \sigma_3$ , a forged target results. But the position of the target is known. It can be seen that the method for solving left/right ambiguity is not perfect, but efficient. There will be three targets contributing to the imaging result of every target in the F-SAR with two receiving antennas.

A similar result for target 3 can be obtained if the azimuth compressed result of  $R_{x1}$  is shifted to the right direction properly and then subjected to the same processing like above. Till now, only the imaging results of sub-area 1 and 3 in the Doppler frequency domain have been evaluated. The imaging result of sub-area 2 also can be obtained with a similar processing method by using the raw data of sub-area 2 from the two receiving antennas.

## 5. Azimuth Resolution and Some Techniques for Implementation

After compression in the range direction and range migration, the step for azimuth compression is a fast Fourier transform (FFT) for the data within the same range gate in one synthetic aperture length  $L$  (or  $N$ ). from the Doppler histories shown in (5), it is known that the Doppler history of the targets in Doppler-time domain is a Basset function. The high resolution results from its correlation pulse is it is

correlated with its matched function. The analysis of the azimuth resolution of the F-SAR from the traditional view of the 3dB width of the correlated pulse is not given in details here [3]. The azimuth resolution is only shown briefly and coarsely from another view.

The length of the synthetic aperture  $L$  is composed from  $N$  discrete points along the moving line of radar platform. If  $N$  points are used for the FFT calculation, the azimuth resolution in Doppler frequency domain is:

$$\delta f = \frac{\text{Prf}}{N} \quad (9)$$

where Prf is the value of the pulse repetition frequency (PRF).

After mapping the Doppler spectrum into the azimuth position  $x$ , the azimuth resolution expressed in the azimuth position distance is:

$$\Delta x \approx \frac{r_0^2 \cdot \lambda}{v \cdot \left| (x_1 + x_2 + \frac{l_1 + l_2}{2}) \right|} \delta f = \frac{\text{Prf} \cdot r_0^2 \cdot \lambda}{N \cdot v \cdot \left| (x_1 + x_2 + \frac{l_1 + l_2}{2}) \right|} \quad (10)$$

where,  $x_1$  and  $x_2$  are the coordinates of two adjacent targets in  $x$  (azimuth) direction and  $\Delta x = |x_1 - x_2|$ .

There are two key techniques which should be considered in the implementation of the F-SAR mode. The first one is the shifting one of the two azimuth compressed data from the two receiving antennas. The frequency difference of the target at the same azimuth position  $x$  to the receiving antennas is:

$$\Delta f(x) = \frac{v \cdot r_0}{\lambda} \left| \frac{1}{\sqrt{(x+l_1)^2 + r_0^2}} - \frac{1}{\sqrt{(x+l_2)^2 + r_0^2}} \right| \quad (11)$$

According to the equation, the shifting can be done precisely.

The second key technique is the mapping at the Doppler into azimuth position. The following equation expresses the relationship between Doppler and its azimuth position.

$$f(x) = \text{mod} \left[ \left( \frac{r_{N-1}(x) - r_N(x)}{\lambda} \cdot \text{Prf} \right), \text{Prf} \right] \quad (12)$$

Where,

$$r_N(x) = \sqrt{x^2 + (R_0 + \frac{N}{2} \cdot v / \text{Prf})^2} + \sqrt{(x+l)^2 + (R_0 + \frac{N}{2} \cdot v / \text{Prf})^2} \quad (13)$$

$$r_{N-1}(x) = \sqrt{x^2 + (R_0 + (\frac{N}{2} - 1) \cdot v / \text{Prf})^2} + \sqrt{(x+l)^2 + (R_0 + (\frac{N}{2} - 1) \cdot v / \text{Prf})^2} \quad (14)$$

Mod( $x, y$ ) means  $x$  modulus  $y$ . And  $f(x)$  means frequency at azimuth position  $x$ . According to (12), the mapping from Doppler domain to azimuth position also can be done precisely.

## 6. Simulation Results

The adopted simulation parameters are listed as following:

- $N=8192 (L=1.0923\text{km})$
- $v=200\text{m/s}$
- $l_1=l_2=10\text{m}$

- $r_1=10km$
- $Prf=1500Hz$
- $\lambda=0.003m$
- $H=2200m$

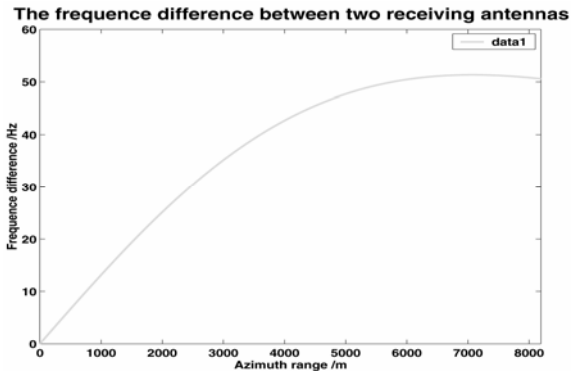


Fig. 4

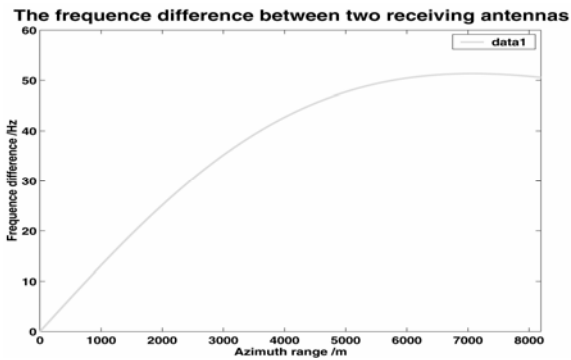


Fig.5 Doppler frequency difference between  $R_{x1}$  and  $R_{x2}$  relative with azimuth position.

The azimuth resolution at different azimuth positions can be calculated with the above simulation parameters. At azimuth position  $x=-1.3km$  or  $1.3km$ , the azimuth resolution is about  $\Delta x=10cm$  according to equ.(9). And at  $x=-50m$  or  $50m$ , the azimuth resolution is about  $\Delta x=2.5m$ . At  $x=0$ , the  $\Delta x \approx 7m$ . The azimuth resolution is changing from about 10cm to 7m with the azimuth position  $x$  from 1.3km to 0. The distance  $l=l_1+l_2$  between  $R_{x1}$  and  $R_{x2}$  affects only slightly the azimuth resolution of targets with large values of the position  $|x|$ , but it affects the azimuth resolution near to  $x=l_1/2$  or  $l_2/2$  very heavily.

The graph of the azimuth resolution is shown in Fig.4 according the above simulation parameters. The Doppler shift between  $R_{x1}$  and  $R_{x2}$  is shown in Fig.5, based on the above simulation parameters.

The whole raw data within one range gate is separated into 32 sub-sections in Doppler domain in this simulation. The matched reference functions of these 32 sub-sections for the azimuth compression are shown in Fig.6.

For 12 point targets within the same range gate  $r_0=10km$  at azimuth positions:  $-1250m < x < -150m$ ,

the distance between any two adjacent targets is  $\Delta d=100m$ , and  $\sigma_r=1$ , without noise; the raw data and azimuth compressed results are shown in the Doppler domain in Fig.7(a). And its result after shifting and multiplying and square root calculation is shown in Fig.7(b).

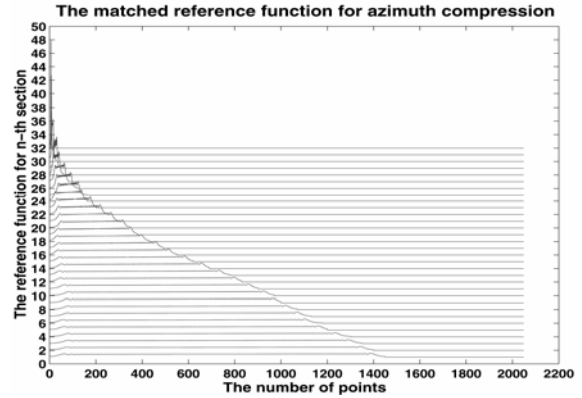
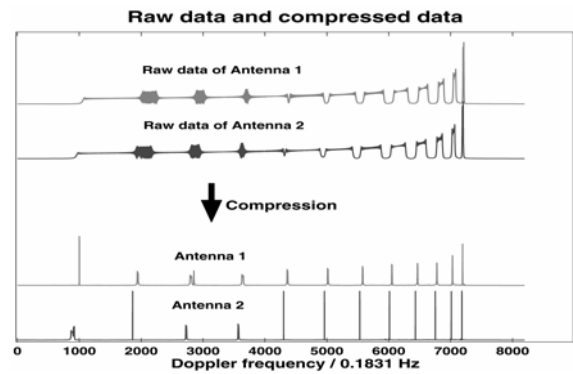
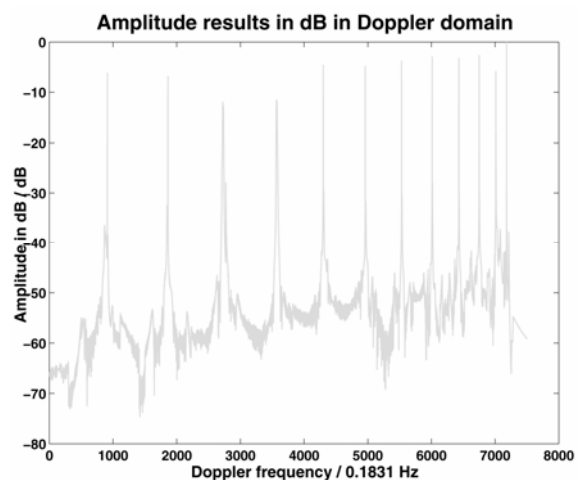


Fig.6 Matched azimuth reference functions for 32 sub-sections



(a)



b)

Fig.7 Simulation result 1: Azimuth compression

For 32 point targets within the same range gate  $r_0=10km$  at azimuth positions:  $-600m < x < -50m$ ,  $\Delta d=50m$ ;  $-890m < x < -780m$ ,  $\Delta d=10m$ ;

$-1320m < x < -1250m$ ,  $\Delta d = 10m$ ; and  $\sigma_i = 1$ , with normalized white noise ( $E=0$ ,  $\sigma=1$ ); the raw data and azimuth compressed results are shown in the Doppler domain in Fig.8(a). And its result after shifting and multiplying and square root calculation is shown in Fig.8(b).

The simulation results shown above are all without ambiguity targets. For 6 point targets within the same range gate  $r_0=10km$  at azimuth positions:  $-50, -100, -150, -200, -250, -300m$  with their symmetrical points at azimuth positions:  $40, 90, 140, 190, 240, 290m$ , the raw data and azimuth compressed results are shown in Doppler domain in Fig.9(a). And the result of target in sub-area I is shown in Fig.9(b) after shifting and multiplying and square root calculation.

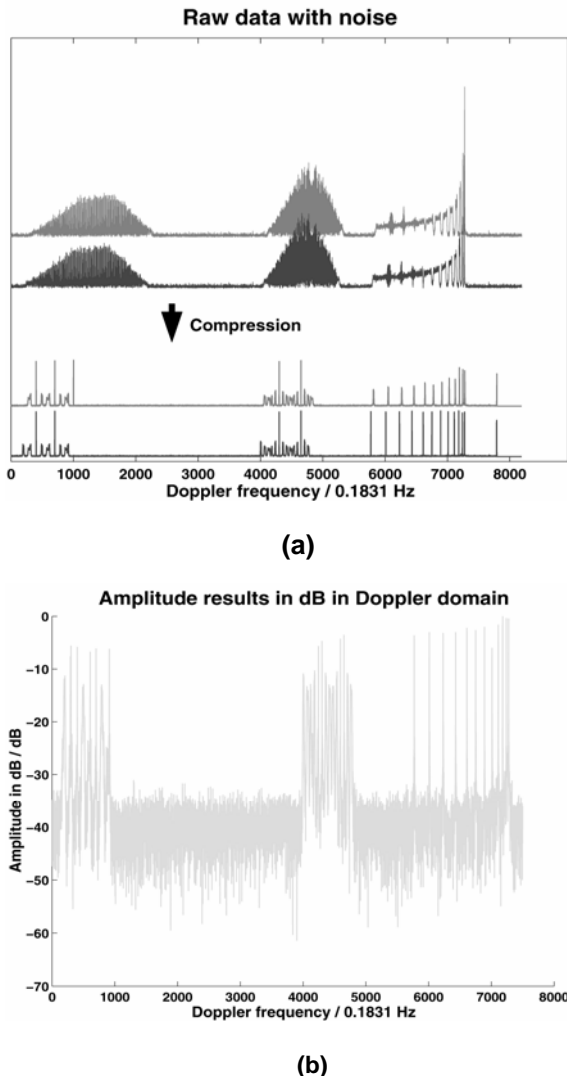


Fig.8 Simulation result 2: Azimuth compression

The simulation results shown above are based on isolated targets. For continuous length target, there is a limitation for the value of maximum length of targets in order to avoid ambiguity. The value of

maximum length in the F-SAR mode with two receiving antennas is  $2l$ . One can see here that it is better to solve the ambiguity if  $l$  is larger. If multiple (more than two) receiving antennas are applied in a F-SAR system, greater target along azimuth direction could be resolved.

From the previous simulation results, it can be seen that the ambiguity problem has been solved, and a good azimuth resolution (from centimeters to several meters) can be obtained by using the proposed system configuration and processing method.

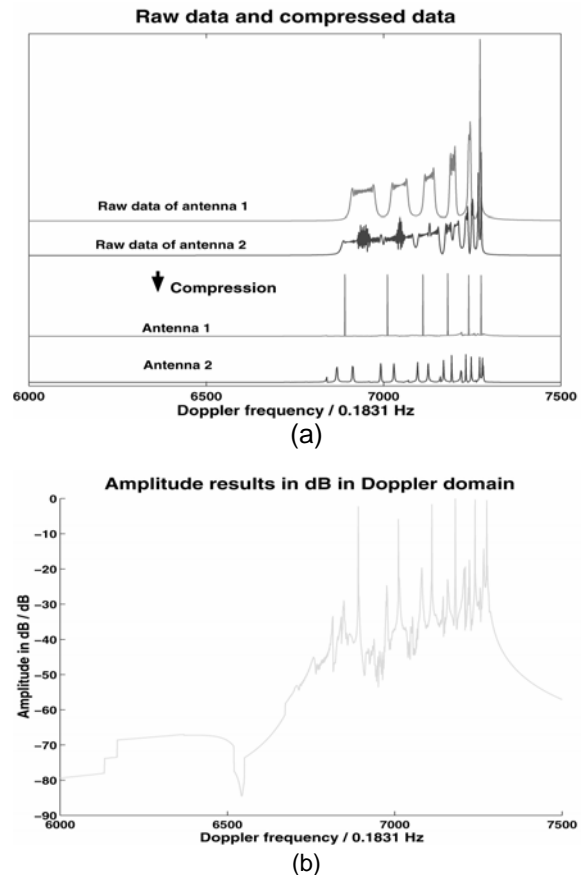


Fig.9 Simulation result 3: Solving the ambiguity

## 7. CONCLUSION

The new imaging mode of F-SAR with two receiving antennas can solve the left/right ambiguity problem of the traditional forward looking radar of F-SAR with one receiving antenna. A high azimuth resolution can be obtained by using the new azimuth compression method proposed in the paper, although the gradient of Doppler frequency is much smaller than that of the side-looking SAR. Wide targets also have been simulated and analyzed in the paper. The resolution of azimuth wide targets is limited to an extension of twice times the separation of the receiving antennas. The analysis and simulation results verified the validity

of the new system configuration and image processing method for the F-SAR proposed in this paper.

Further work on the F-SAR has to solve the experimental system implementation. For the F-SAR with multiple receiving antennas, the theory and processing method are similar with that of the imaging mode of this paper. The future theory will be focused on the optimization of the azimuth reference functions to acquire perfect imaging results, and the F-SAR with multiple receiving antennas. The compensation of range migration for the image processing of the F-SAR mode is also a future work.

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