Non-Diffusive Lateral Transport on the Moon: the Importance of High Velocity Ejecta

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ABSTRACT High spatial resolution mapping of compositional gradients across basalt-highland contacts on the Moon with Clementine UV-VIS multispectral data reveals a highly symmetric distribution of basalt and highland across the geological contacts. Thus lateral transport dominates over deep vertical transport during the formation of the observed mixing zones. Because the process of repetitive meteorite bombardment governs lateral transport across the geological contacts, classical diffusion models have been invoked to model this lateral transport, but are shown to be inadequate. The high velocity component of impact ejecta travels long distances, following a -3 power decay law, which results in an infinite variance of the average particle transport. An anomalous diffusion model was developed that accommodates the infinite variance. This result indicates that high velocity crater ejecta dominates the lateral transport of material on the Moon.

KEY WORDS Lateral Transport, Clementine, Nonlinear Mixture, Anomalous Diffusion, High velocity ejecta.

1. Introduction

Impact cratering is considered the primary process leading to vertical and lateral mixing in the lunar regolith (Gault et al., 1974; Gold and Williams, 1974; Langevin et al., 1977; Hörz, 1978). However, the relative importance of vertical vs. lateral material transport on the Moon has been intensely debated for over 30 years. A number of lines of evidence derived from theoretical impact assessments (Shoemaker et al., 1970; Arvidson et al., 1975), analysis of geochemical gradients across mare-highland contacts (Adler et al., 1974; Rhodes, 1977), and petrographic data of lunar samples and cores (Laul et al., 1981; Simon et al., 1990) have been used to argue that vertical transport dominates over lateral transport. Conversely observations of crater ejecta facies, theoretical calculations and experiments suggest lateral transport is more important than vertical transport (Schultz and Gault, 1985).

Impact cratering is assumed to be random in space and time. Thus the transport path of an individual grain would be analogous to a random walk (Borg et al., 1971; Duraud et al., 1975), and amenable to modeling by classical diffusion equations (Chandrasekhar, 1943). Diffusion models were successfully developed for vertical transport in the lunar regolith (Blake and Wesserberg, 1975, Langevin et al., 1982), but were never adopted for lateral transport because of a lack of high precision data of material abundance across extensive areas of the Moon to constrain and validate models. Though lateral transport was viewed by many researchers to be important, this lack of data left the debate unresolved.

2. New Observations with Clementine Data

In 1994, the Clementine spacecraft orbited the Moon and imaged the 90% lunar surface with the UV-VIS multispectral camera with a typical spatial resolution of 150 meters (Nozette et al., 1994). These data have the necessary spatial and spectral characteristics to reexamine this debate.

For this analysis we have focused on the contact between mare and highland along the southern edge of the Grimaldi Basin on the western near-side of the Moon (Fig. 1). This site was selected because it exhibited few post-mare perturbations to the contact and was free of impact craters larger than a few 100 meters in size. The multispectral image data were analyzed with a nonlinear spectral mixture model to produce estimates of the surface abundance of mare and highland (Mustard and Pieters, 1989; Mustard et al., 1998) for each pixel in the image to a precision of ± 1.5%. Profiles of material abundance orthogonal to the geological contacts were extracted for analysis. Shown in Fig. 2 is a profile which was created from 85 profiles averaged in the direction parallel to the contact.

Based on the analysis of this and other profiles, the following critical observations are made: i) we identify two distinct mixing zones on each side of the geological contact. A steep gradient in mare abundance occurs near the mare-highland contact, while a smaller gradient occurs far from the contact. The steep mixing zone averages ≈4 km in width with a gradient of

≈13 %/km, and the moderate mixing zone is ≈25 km in width and has a gradient of <2%/km; ii) We observe a highly symmetric distribution of material across the contacts in all the profiles with mare transport to the highland coupled to highland transport to the mare; iii) The abundance of mare and highland are equal to 50% at the geological contact (Mustard and Pieters, 1989; Mustard et al., 1998). These consistent characteristics have been observed for mare-highland boundaries in the Grimaldi, Tsiolkovsky, Orientale and Fecunditatis basins, as well as a mare-mare boundary between Tranquillitatis and Serenitatis (Li and Mustard, 1999). Sources of variation in the observed signal that may be due to instrumental or geomorphic effects have been examined, are negligible (Li et al., 1999). Thus, these basic observations demonstrate that lateral transport dominates the formation of the mixing zones.

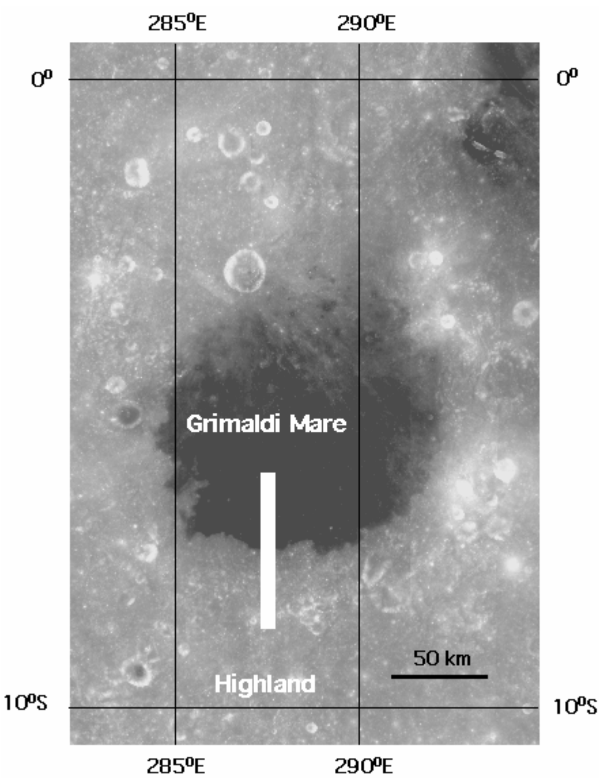


Figure 1: Location of the mare abundance profile shown in Fig. 2 superimposed on a mosaic of Clementine UV-VIS images (750-nm band) of the Grimaldi region.

3. Anomalous Diffusion Model

To investigate the magnitude of lateral transport and its effects on the compositional gradients, a quantitative and physically meaningful model is required to relate the pattern of the observed profiles of material abundance to geological processes. At first glance, the profile of mare abundance in Fig. 2 is very similar to that expected for classical diffusion, and a model based on diffusion principles might be expected given the stochastic nature of impact cratering. However, a properly formulated diffusion model based on impact cratering either a) provide a poor fit or b) requires two diffusion coefficients to fit the observed profiles of mare abundance (Li and Mustard, 1997). The second result, while mathematically valid, isn't physically valid because the random path of the particle and central limit theorem of diffusion theory both imply that a model with only one diffusion coefficient is permitted. It is therefore required to re-evaluate the assumption of diffusion for lateral transport.

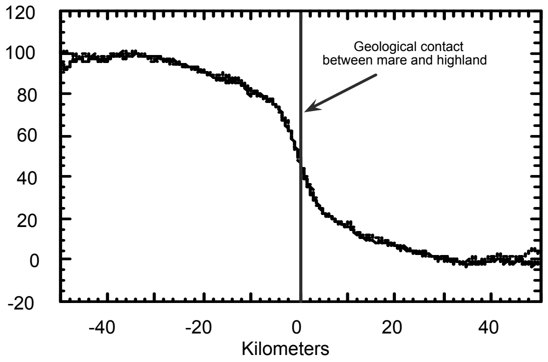


Figure 2: Profile of mare abundance across the geological contact between mare and highland in Grimaldi Basin. The Solid line is the actual abundance profile, while the dotted line shows the same profile but with the distance and abundance axes reversed. This graphically illustrates the high degree of compositional symmetry.

In order to model lateral transport by a stochastic approach such as classical diffusion, the ejecta thickness distribution needs to be transformed into a probability density distribution (PDD) by dividing the ejecta thickness distribution by the total ejecta volume. The ejecta thickness distribution for a simple impact event as a function of distance from the crater scales with crater radius R by the simple power law relationship.

$$h = T\left(\frac{r}{R}\right)^{\alpha} \qquad (1)$$

where h is the ejecta thickness, T is ejecta thickness at the rim, r is the radial distance from crater rim, R is crater radius and the index α is typically given a value of -3. Then the probability density distribution, $p(r)$, is given by:

$$p(r) = \frac{T\left(\frac{r}{R}\right)^{-3}}{2\pi TR^2} \qquad (2)$$

where $2\pi TR^2$ is the total volume of ejecta, subject to the constraint that the integral of $p(r)$ over all crater ejecta equals 1.0 (McGetchin et al., 1973; Stöffler et al., 1975; Schultz, 1999).

Assuming a symmetric distribution of ejecta about the crater, the variance of the displacement of a given ejecta particle in a two-dimensional system is given by:

$$\langle r^2 \rangle = 2\pi \int_0^L r^2 p(r) r dr = \int_R^L R dr \qquad (3)$$

where L denotes the farthest distance of particle's flight. Classical diffusion requires a finite variance for the ejecta PDD. Ejecta from impacts on airless bodies consist of a continuous component, generally within 2.5 radii of the crater rim, and a discontinuous (high velocity) component that may extend many 10's of radii from the rim. If we only consider the continuous ejecta, the variance is finite, lateral transport of a particle is diffusive, and lateral transport by impact can be modeled by a classical diffusion approach. However, the variance will be infinite if we include continuous and discontinuous ejecta (i. e. L is effectively infinite).

The precise distribution and fate of discontinuous ejecta is not well known, but it is capable of traveling great distances and may even escape the gravity field of the Moon (Schneider, 1975; Li and Mustard, 1999) and thus may significantly contribute to the lateral transport of materials (Hörz, 1977). To accommodate the contribution of high velocity ejecta to lateral transport, a classical diffusion model would require a power decay law of ejecta thickness with $\alpha <$ – 4 for the total ejecta thickness distribution. Such a distribution is not consistent with observations of laboratory and natural craters (McGetchin et al., 1973; Stöffler et al., 1975; Schultz, 1999). Including the contribution of high velocity ejecta while also using a –3 power decay for the ejecta thickness distribution causes a infinite variance, and thus lateral transport on the lunar surface due to impact process is non-diffusive. While non-diffusive processes cannot be modeled with classical diffusion, they can be modeled using "anomalous diffusion" (Hughes et al., 1982; Bouchard and Georges, 1990; Tsallis et al., 1995; Klafter et al., 1995). Anomalous diffusion refers to the situation where the mean squared displacement doesn't scale linearly with time as is required for diffusion process (Hughes et al., 1982; Bouchard and Georges, 1990; Tsallis et al., 1995; Klafter et al., 1995).

We have derived a model for non-diffusive transport of impact ejecta based on the concepts of anomalous diffusion. First we derive the characteristic function for the ejecta PDD (Eq. 2) by conducting a two-dimensional Fourier Transformation. This characteristic function is only for a single displacement of a particle. The actual transport of the particle on the lunar surface results from multiple cratering events in which the number of times a particle is displaced obeys a Poisson density distribution (Chapman et al., 1970; Gault et al., 1974). The characteristic function for the probability of the displacement of the particle caused by these multiple cratering events can be calculated using convolution theorem in Fourier space (Bochner and Chandasekharan, 1949). The inverse Fourier Transformation of this characteristic function leads to a new PDD for multiple cratering events which defines the mare abundance at given location for a point source. To obtain the mare abundance for a plane source in one-dimensional space (i. e. orthogonal to the geological contact), we integrate the mare abundance for a series of the point sources along the geological contact. Finally we derive the mare abundance for a half-space source by summing the mare abundance contributed by the plane sources of infinite number on the mare side. This leads to the basic formula:

$$C(x,t) = 50 - 31.83 \arctan\left(\frac{x}{A}\right) \qquad (4)$$

$$A = \pi K(t) D_{\min}^{-0.4} \qquad (5)$$

where C is the mare abundance at the distance x from the geological contact at time t, $D_{\min}$ is the minimum diameter of craters excavating the particle, and K is a parameter describing the size-frequency distribution of crater population (Shoemaker et al., 1970).

4. Results and Implications

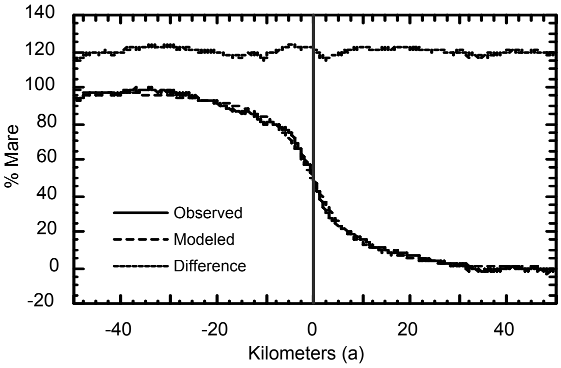
Fig. 3a shows observed and modeled profiles of mare abundance across the mare-highland boundary in the Grimaldi Basin.

The modeled profiles are derived by fitting Eq. (4) to observed data and solving for A. It is evident that the fit of the model to the observed profile is reasonable. The improvement of the model in fit can be appreciated by comparing to the fit of the diffusion model shown in Fig. 3b. The anomalous diffusion model fits the steep and moderate mixing zones simultaneously, while the diffusion model fails to fit the steep mixing zone.

This result has also be repeated for compositional profiles extracted from the Tsiolkovsky, Orientale and Fecunditatis basins.

The distribution of mare across a compositionally distinct boundary created by lateral transport by impact cratering is well modeled by only an elementary function derived from the anomalous diffusion model. This model fits the compositional variations much better than the diffusion model does. The significance of this result can be appreciated in the following.

First, the result indicates that the high velocity ejecta typical of discontinuous ejecta, though small in volume for any given impact, is very important to lateral transport. Any models for lateral transport due to impact must accommodate the contribution of high velocity ejecta. Although many models of regolith dynamics have been developed (Langevin et al., 1977), only a few of them directly address lateral transport of the regolith. Those models that do address lateral transport ignore the effects of multiple displacements and the consequences for particle transport (Arvidson, 1975), hence predicting inefficient lateral transport process though formulated with the same assumption and conditions we used. Shoemaker et al. (1970) developed a model dealing with impact into solid rock, but not regolith. A numerical Monte Carlo model that accommodated multiple impacts was developed for understanding lateral mixing of the regolith (Arnold, 1975a; Arnold, 1975b). However, this and the Shoemaker et al. (1970) model only accommodate ejecta deposits within the range of 1-4 crater diameters, and thus ignored lateral transport due to high velocity ejecta.



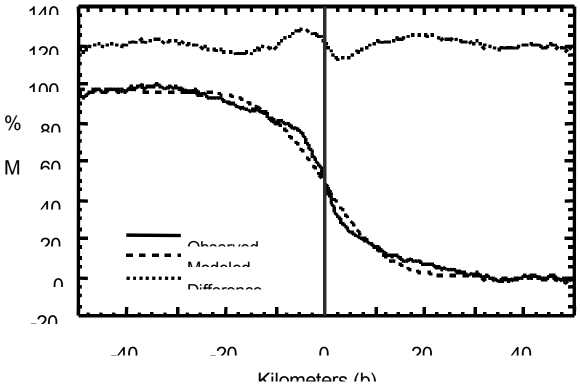


Figure 3: Observed and modeled abundance determined using (a) the anomalous diffusion model and (b) a classical diffusion model. In each the solid line is the observed profile, the dashed line is the modeled, and the dotted line shows the difference between the observation and modeled.

Second, the result strongly supports our conclusion that lateral transport dominates the formation of compositional gradients across geological contacts on the Moon. The essential elements of non-diffusive transport (i. e. infinite variance and multiple displacements) are the keys to the importance of the lateral transport process. If vertical transport dominates over lateral transport and secondary cratering efficiency is high (Oberbeck, 1975), the contribution of high velocity primary ejecta can be ignored. In this case, the total primary ejecta volume would thus distribute within a finite range from a crater center, the variance of the total ejecta is finite, and a model formulated with the classical diffusion principle would be adequate. Conversely, the failure of diffusion model to fit the compositional profiles indicates that vertical transport is unimportant.

Our basic conclusions that lateral transport is an efficient process on the Moon, driven primarily by the high velocity component of impact ejecta is derived from three recent developments. 1) Clementine global multispectal imaging data were acquired with sufficient spatial and compositional resolution to assess modes of material transport. 2) Nonlinear spectral mixture modeling (Mustard and Pieters, 1989; Mustard et al., 1998) provides an approach to quantitatively extract the material abundance from these data. These data sources and the analytical approach were not previously available for extracting the petrological material abundance across compositionally distinct units on the Moon. 3) The concept of anomalous diffusion with infinite variance was not applied to dynamical modeling in physics until the 1990s though its basic idea had been proposed in 1920s by Lévy (Bouchard et al., 1990). The lack of the mathematical techniques for modeling anomalous diffusion with infinite variance prevented the development of the model for lateral transport that accommodated the entire range of ejecta velocities.

An important implication of our analysis lies in its application to dating craters and investigating the cratering rate. We assume that lateral transport of materials across the lunar surface originates through repetitive meteorite impact. Thus, the variation of material abundance must relate to the ages of surface features and the meteorite flux, and the derived relationship for A can be extended for application to the determination of meteorite flux and relative age of surfaces. If a constant cratering rate is assumed to be responsible for the lateral transport of materials, A is directly proportional to K and t; if the cratering rate has not been constant, then A is directly proportional to K only, and both A and K depend nonlinearly on t. The nonlinear relationship between A and t can be derived by investigating craters with various known ages, then K can be related to t. This remote dating method can be a complimentary approach to the use of spectral parameterization of soil maturity (Grier et al., 1999). The latter is limited by saturation of spectral parameters and material composition (Staid and Pieters, 1999).

5. Conclusions

In summary, new observations of the variation of mare and highland abundances across simple geological contacts indicate that lateral transport dominates over vertical transport during the regolith mixing on the lunar surface. Due to the wide dispersal of ejecta, a classical diffusion model is not valid to model this lateral transport. However, an anomalous diffusion model can accommodate the total ejecta volume and specifically accommodate the contribution of high velocity ejecta. High velocity ejecta is then identified as a very important component of lateral transport.

References

Adler, I., M. Podwysocki, C. Abdre, J. Trombka, E. Eller, R. Schmadebeck, and Y. Yin, 1974, The role horizontal transport as evaluated from the Apollo 15 and 16 orbit experiments, *Proc. Lunar Sci. Conf. 6th,* 975-979.

Arnold, J. R.,1975a, Monte Carlo model for the gradening of the lunar regolith, *Moon*., 8:35-24.

Arnold, J. R., 1975b, Monte Carlo simulation of turnover processes in the lunar regolith, *Proc. Lunar Sci. Conf. 6th,* 2375-2395.

Arvidson, R. E., R. O. Drozd, and C. M. Hohenberg, C. J. Morgan, and G. Poupeau, 1975, Horizontal transport of the regolith, modification of features, and erosion rates on the lunar surface, *Moon*, 13: 67-79.

Blake, M. J., and G. J. Wasserberg, 1975, Cratering and cosmogenic nuclides, *Geophys. Res. Lett.*, 2:477-479.

Bochner, E. and K. Chandasekharan, 1949, *Fourier Transformation*, Princeton University Press, New Jersey, Chapter 2.

Borg, J., G. M. Comstock, Y. Langevin, and M. Maurette, 1976, A Monte Carlo model for the exposure history of lunar dust grains in the ancient solar wind, *Earth and Planet. Sci. Lett.*, 29: 161-174.

Bouchard, J. P. and A. Georges, 1990, Anomalous diffusion in disordered media: Statistical mechanism, model and physical application, *Phys. Rep.,* 195: 127-293.

Chandrasekhar, 1943, Stochastic problems in physics and astronomy*, Rev. Modern Phys.*, 15: 1-87.

Chapman, C. R., J. A. Mosher, and G. Simmons, 1970, Lunar cratering and erosion from Orbiter 5 photographs, *J. Geophys. Res.*, 75: 1445-1466.

Duraud, J. P., Y. Langevin, and M. Maurette, 1975, The simulated depth history of dust grains in the lunar regolith, *Proc.Lunar Sci. Conf. 6th,* 2397-2415.

Gault, D. E., F. Hörz, D. E. Brownlee, and J. B. Hartung, 1974, Mixing of the lunar regolith, *Proc. Lunar. Sci. Conf.5th* , 2365-2386.

Gold, T. and G. J. Williams, 1974, On the exposure history of the lunar regolith, *Proc. Lunar Planet Sci. Conf. 5th*, 2387-2345.

Grier J. A., A. S. McEwen, P. G. Lucey, R. G. Strom, M. Milazzo, 1999, Relative ages of large rayed lunar craters: Implications, *In Lunar and Planetary Science XXX, Abstract # 1910*, Lunar and Planetary Institute, Houston (CD-ROM).

Hörz, F., 1978, How thick are mare basalts ?, *Proc. Lunar Planet Sci. Conf. 9th*, 3311-3331.

Hörz, F., 1977, Impact cratering and regolith dynamics, *Phys. Chem. Earth, X: 3-15*, Pergamon Press.

Houston, W. N., and Y. Moriwaki, C.-S. Chang, 1973, Downslope movement of lunar soil and rock caused by meteoroide impact, *Proc. Lunar Sci. Conf. 4th*, 2425-2435.

Hughes, B. D., E. M. Montroll and M. F. Shlesinger, 1982, Fractal random walks*, J. Stat. Phys*., 28: 111-125.

Klafter, J., G. Zumofen, and M. F. Shlesinger, 1994, Lévy description of anomalous diffusion in dynamical system, *In Lévy Flight and Related Topics in Physics (Eds. M. F. Schlesinger and G. M. Zaslavsky)*, 195-215.

Langevin, Y., J. R. Arnold, 1977, The evolution of the lunar regolith, *Ann. Rev. Earth Planet Sci,* 5: 449-89.

Langevin, Y., J. R. Arnold and K. Nishiizumi, 1982, Transport processes on the lunar surface: Comparison of model calculations with radionuclides data, *J. Geophys. Res*., 87: 6681-6691.

Laul, J. C., J. J. Papike, and S. B. Simon, 1981, The lunar regolith: Comparative studies of the Apollo and Luna sites, chemistry of soils from Apollo 17, Luna 16, 20 and 24, *Proc. Lunar Planet Sci. 12th*, B389-B407.

Li L., J. F. Mustard, and G. He, 1997, Compositional gradients across mare-highland contact: The importance of lateral mixing, *In Lunar and Planetary Science XXVIII*, 811-812, Lunar and Planetary Institute, Houston.

Li L., J. F. Mustard, and C. M. Pieters, 1999, The effects of scattered light in the Clementine UV-VIS camera on mixture analysis, *In Lunar and Planetary Science XXX, Abstract #1866*, Lunar and Planetary Institute, Houston (CD-ROM).

Li L., and J. F. Mustard, 1999, Lateral transport of materials across the lunar surface: An anomalous diffusion model, *In Lunar and Planetary Science XXX, Abstract # 2012*, Lunar and Planetary Institute, Houston (CD-ROM).

McGetchin T. R., M. Settle, J. W. Head, 1973, Radial thickness variation in impact crater ejecta: Implications for lunar basin deposits, *Earth Planet Sci. Lett*., 20: 226-236.

Mustard, J. F. and C. M. Pieters, 1989, Photometric phase functions of common geologic minerals and applications to quantitative analysis of mineral mixture reflectance spectra, *J. Geophys. Res.*, 94: 13,619-13,634.

Mustard, J. F., L. Li, and G. He, 1998, Nonlinear spectral mixture modeling of lunar multi-spectral: Implications for lateral transport, *J. Geophys. Res*., 103; 19,519-19,425.

Nozette S. et al., 1994, The Clementine Misssion to the Moon: Scientific overview, *Science*, 266: 1835-1839.

Oberbeck, V. R., 1975; The role of ballistic erosion and sedimentation in lunar stratigraphy, *Rev. Geophys.*, 13: 337-362.

Rhodes, J. M., 1977, Some compositional aspects of lunar regolith evolution, *Philos.Trans. R. Soc. London* , A285: 293-301.

Schultz, P. H., and D. Gault, 1985, Clustered impacts : Experiments and implications, *J. Geophys. Res*., 90: 3701-3732.

Schultz, P. H., 1999, Ejecta distributions from oblique impacts into particulate targets, *In Lunar and Planetary Science XXX, Abstract # 1919*, Lunar and Planetary Institute, Houston (CD-ROM).

Schneider, E., 1975, Impact ejecta exceeding lunar escape velocity, *Moon,* 13: 173-184.

Shoemaker, E. M., M. H. Hait, G. A. Swann, D. L. Schleicher, G. G. Schaber, R. L. Sutton, D. H. Dahlem, E. N. Goddard, A. C. Waters, 1970, Origin of lunar regolith at Tranquility base, *Proc. Apollo 11 Lunar Sci. Conf.*, 3; 2399-2412.

Simon, S. B., J. J. Papike, D. C. Gosselin, J. C. Laul, S. S. Hughes, and R. A. Schmitt, 1990, Petrology and chemistry of Apollo 17 regolith breccias: A history of mixing of highland and mare regolith, *Proc. Lunar Planet Sci. Conf. 20th*, 219-230.

Staid, M. I., C. M. Pieters, and J. W. Head, 1999, Integrated spectral analysis of mare soils and craters: Characterization of lunar basalt types, In Lunar and Planetary Science XXX, Abstract # 1724, Lunar and Planetary Institute, Houston (CD-ROM).

Stöffler, D., D. E. Gault, J. Wedekind and G. Polkowski, 1975, Experimental hypervelocity impact into quartz sand: Distribution and shock metamorphism of ejecta, *J. Geophys. Res.,* 80: 4062-4077.

Tsallis, C., A., M. C., de Souza and R. Maynard, 1995, Derivation of Lévy-type anomalous super-diffusion from generalized statistical mechanics, *In Lévy Flight and Related Topics in Physics (Eds. M. F. Schlesinger and G. M. Zaslavsky),* 269-287.